



## Rate regions for coordination of Decode-and-Forward relays and direct users

Thai, Chan; Popovski, Petar

*Published in:*  
2012 IEEE International Conference on Communications (ICC)

*DOI (link to publication from Publisher):*  
[10.1109/ICC.2012.6364628](https://doi.org/10.1109/ICC.2012.6364628)

*Publication date:*  
2012

*Document Version*  
Early version, also known as pre-print

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*  
Thai, C., & Popovski, P. (2012). Rate regions for coordination of Decode-and-Forward relays and direct users. In *2012 IEEE International Conference on Communications (ICC)* (pp. 4468-4472). IEEE Communications Society. I E E E International Conference on Communications <https://doi.org/10.1109/ICC.2012.6364628>

### General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal -

### Take down policy

If you believe that this document breaches copyright please contact us at [vbn@aub.aau.dk](mailto:vbn@aub.aau.dk) providing details, and we will remove access to the work immediately and investigate your claim.

# Rate Regions for Coordination of Decode-and-Forward Relays and Direct Users

Chan Dai Truyen Thai and Petar Popovski  
Department of Electronic Systems, Aalborg University  
Email:{ttc, petarp@es.aau.dk}

**Abstract**—Recently, the ideas of wireless network coding (NC) has significantly enriched the area of wireless cooperation/relaying. They bring substantial gains in spectral efficiency mainly in scenarios with two-way relaying. Inspired by the ideas of wireless NC, recently we have proposed techniques for coordinated direct/relay (CDR) transmissions. Leveraging on the fact that the interference can be subsequently canceled, these techniques embrace the interference among the communication flows to/from direct and relayed users. Hence, by allowing simultaneous transmissions, spectral efficiency is increased. In our prior work, we have proposed CDR with Decode-and-Forward (DF) relay in two scenarios. In this paper, we extend the two existing regenerative CDR schemes and proposed for the other two scenarios such that all schemes benefit from the aforementioned principle of containing the interference. The parameters in the schemes are optimized to have the largest rate region or the highest sum-rate. Numerical results show that DF CDR is better than the reference scheme and almost better than AF CDR.

**Index Terms**—Cooperative communications, relaying, analog network coding, interference cancelation, a priori information.

## I. INTRODUCTION

Recently there have been extensive studies on cooperative, relay-based transmission schemes for extending cellular coverage or increasing diversity. Several basic relaying transmission techniques have been introduced, such as amplify-and-forward (AF) [1], decode-and-forward (DF) [2] and compress-and-forward (CF) [3]. These transmission techniques have been applied in one-, two- or multi-way relaying scenarios.

In particular, two-way relaying scenarios [4], [5], [6] have attracted a lot of attention, since it has been demonstrated that in these scenarios one can apply techniques based on NC in order to obtain a significant throughput gain. There are two basic principles used in designing throughput-efficient schemes with wireless NC (1) *aggregation of communication flows* - NC operates by having the flows sent/processed jointly; (2) *intentional cancelable interference*: flows are allowed to interfere over the wireless channel, knowing *a priori* that the interference can be cancelled by the destination.

Using these insights, in [7] we have proposed schemes, depicted on Figs. 1 and 2 for traffic scenarios that are more general than the usual two-way relaying. These schemes are termed *coordinated direct/relay (CDR)* transmissions. In the scheme on Fig. 1, termed  $S_1$ , U receives downlink traffic from the BS, while V sends uplink traffic to the BS. For the scheme  $S_1$  (Fig. 2, these traffic patterns are inverted), in the first step the BS transmits to the relay RS. In the second step, RS

transmits to U and simultaneously V transmits to the BS. The reception of V's signal at BS is interfered by the transmission of RS; however, since BS knows the signal of RS *a priori*, it can cancel it and get a "clean" message from V. Enabling such simultaneous transmissions improves the spectral efficiency. In scheme  $S_2$ , in the first step BS sends to V and simultaneously U sends to RS, such that RS receives interference of these two signals, such as in analog NC for two-way relaying. But, unlike two-way relaying, the signal sent by RS in the second step need only be decoded at BS, but not at U. This makes the link RS-U irrelevant and, as we will see later, deflecting the traffic to go BS-V instead of BS-RS-U, and combining it with the traffic U-RS-BS, can give advantages in the sum-rate.

We have considered RS that uses Amplify-and-Forward (AF) in [8] and proposed two schemes of Decode-and-Forward (DF) for two CDR scenarios in [9]. In this paper we extend the two existing schemes and proposed DF CDR schemes for the other two CDR scenarios such that in all schemes, a station uses the information about the interference to cancel it and decode the desired signal. The choice of the duration of different phases in the schemes  $S_1$ ,  $S_2$ ,  $S_3$  and  $S_4$  is subject to optimization. The optimization objective is the rate region and the sum-rate for each of the respective schemes.

The rest of the paper is organized as follows. Section II introduces the system model. The DF reference and CDR schemes are described in section III. Section IV shows and discusses some numerical results. Section V concludes the paper.

## II. SYSTEM MODEL

We consider a scenario with one base station (BS), one relay (RS), and two users (U and V), see Fig. 1. All transmissions have a unit power and normalized bandwidth of 1 Hz. Each of the complex channels  $h_i, i \in \{1, 2, 3, 4\}$ , is reciprocal, known at the receiver. All the channels are known at BS.

In the scenario, BS sends messages  $s_1$  to U and  $s_4$  to V and receives messages  $s_3$  from U and  $s_2$  from V. Note that the example on Fig. 1 does not show traffic patterns that involve  $x_3$  and  $x_4$ , but they are used on Fig. 2. We assume that the data to/from each user is *infinitely backlogged* so that there is always data to transmit [10]. In each scheme, depending on the channel status, message  $s_i, i \in \{1, \dots, 4\}$  can be divided into sub-messages  $s_{i,1}$  and  $s_{i,2}$ . If message  $s_i, s_{i,1}$  or  $s_{i,2}$  is sent from BS, U or V, it is encoded to symbol string  $x_i, x_{i,1}$  or  $x_{i,2}$  respectively. If it is sent from RS we have the symbol

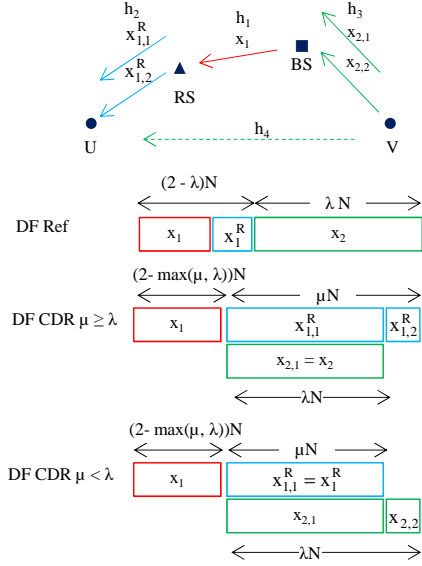


Fig. 1. Time slots in Reference and CDR  $S_1$  Schemes.

string  $x_i^R$ ,  $x_{i,1}^R$  or  $x_{i,2}^R$  respectively. Denoting  $|s|$  and  $|x|$  as the number of bits and number of symbols of message  $s$  and symbol string  $x$  respectively, we have  $|s_i| = |s_{i,1}| + |s_{i,2}|$  and  $|x_i| = |x_{i,1}| + |x_{i,2}|$ . Because there are many cases of channels are considered, we combine some similar cases and describe the schemes in the combined one. Therefore, if  $s_{i,2}$  and  $x_{i,2}$  are not mentioned in a scheme, it means that  $|s_{i,2}| = 0$ ,  $|x_{i,2}| = 0$  and  $s_{i,1} = s_i$ ,  $x_{i,1} = x_i$ .

The direct channel BS–U is assumed weak and U gets the information from BS only through the decoded/forwarded signal from RS. If in slot  $k$ , the reception of  $x$  at node  $m$  is additionally interfered by  $w$ , then the received signal is  $y_m[k] = h_i x + h_j w + z_m[k]$ ,  $k \in \{1, 2, 3\}$ ,  $m \in \{B, R, U, V\}$  where  $z_m[k] \sim \mathcal{CN}(0, \sigma^2)$  is Additive White Gaussian Noise (AWGN). Denoting the capacity function as  $C(\gamma) = \log_2(1 + \gamma)$ , we can write the capacity of such a transmission as  $C_{i-j} = C\left(\frac{|h_i|^2}{|h_j|^2 + \sigma^2}\right) = C\left(\frac{g_i}{g_j + 1}\right)$  with  $g_i = \frac{|h_i|^2}{\sigma^2}$ . In case there is no interfering signal the capacity is  $C_i = \log_2(1 + g_i)$ . If the receiver jointly decodes  $x$  and  $w$ , the maximal sum rate for these two signals is  $C_{ij} = \log_2(1 + g_i + g_j)$ . It is straightforward to see that  $C_{ij} = C_{ji}$ ,  $C_{ij} = C_j + C_{i-j}$ .

In each scheme, the total time length is  $2N$  symbols.  $R_U^i$  and  $R_V^i$ ,  $i \in \{E, S_1, S_2, S_3, S_4\}$  are maximal rates for U and V respectively in scheme  $i$ .  $E$  denotes the reference scheme, all schemes will be described in the next part. The sum-rate is therefore estimated as  $R_S^i = R_U^i + R_V^i = \frac{1}{2N}(D_U^i + D_V^i)$ , where  $D_U^i, D_V^i$  represent the corresponding number of bits. The transmission for the direct user has a duration of  $\lambda N$  symbols. In the following part, we analyze the choice of  $\lambda$  with respect to the optimization of the sum-rate.

### III. REFERENCE AND CDR SCHEMES

CDR scheme 1 denoted as  $S_1$  delivers two messages  $s_1$  and  $s_2$ . CDR schemes  $S_2$ ,  $S_3$  and  $S_4$  deliver messages pairs  $(s_3, s_4)$ ,  $(s_1, s_4)$  and  $(s_2, s_3)$  respectively. CDR schemes combine the transmissions of the two messages in such a

way that the information about the interference is exploited as much as possible while reference schemes use orthogonal transmissions by multiplexing them in time. However, since the transmit power of all nodes are the same and all channels are reciprocal, 4 reference schemes which are corresponding to 4 CDR schemes have the same rates.

#### A. Reference Scheme

First, BS encodes  $s_1$  to  $x_1$  with rate  $R_1$  and transmits it to RS  $y_R[1] = h_1 x_1 + z_R[1]$ . Second, RS decodes  $x_1$  to  $s_1$ , re-encodes it to  $x_1^R$  with rate  $R_1^R$  and transmits it to U (see Fig. 1)  $y_U[2] = h_2 x_1^R + z_U[2]$ . Third, V encodes  $s_2$  to  $x_2$  with rate  $R_2$  and transmits it to BS  $y_B[3] = h_3 x_2 + z_B[3]$ . Since the V–BS transmission's length is pre-defined as  $\lambda N$  symbols and all transmissions are performed separately, the total time length for U is therefore  $(2 - \lambda)N$ . We denote the number of symbols in the RS–U transmission as  $\mu N$ . The rates  $R_1$ ,  $R_1^R$  and  $R_2$  are selected as the maximal rates over the corresponding channels  $R_1 = C_1$ ,  $R_1^R = C_2$  and  $R_2 = C_3$ . The maximal data sent through the BS–RS, RS–U and V–BS transmissions are respectively  $D_{U_1}^E = (2 - \lambda - \mu)NC_1$ ,  $D_{U_2}^E = \mu NC_2$ ,  $D_V^E = \lambda NC_3$ . The total data transmitted for two users is  $D_S^E = \min(D_{U_1}^E, D_{U_2}^E) + D_V^E$ . Since  $D_V^E$  does not depend on  $\mu$ ,  $D_{U_1}^E$  is a decreasing function and  $D_{U_2}^E$  is an increasing function of  $\mu$ , in order to get maximal  $D_S^E$ ,  $\mu$  is selected such that  $D_{U_1}^E = D_{U_2}^E$ . Solving this equation we have the optimal  $\mu = \mu_{\text{opt}}^E = \frac{(2-\lambda)C_1}{C_1+C_2}$ . The data for U and V are respectively  $D_U^E = (2 - \lambda)N \frac{C_1 C_2}{C_1 + C_2}$ ,  $D_V^E = \lambda NC_3$ . The sum-rate is  $R_S^E = \frac{(2-\lambda)C_1 C_2}{2(C_1 + C_2)} + \frac{\lambda NC_3}{2}$ .

#### B. CDR Scheme 1

First, BS transmits  $x_1$  to RS (see Fig. 1)  $y_R[1] = h_1 x_1 + z_R[1]$ . Second, RS decodes  $x_1$  to  $s_1$ , divides it into two sub-messages, re-encodes them to  $x_{1,1}^R$ ,  $x_{1,2}^R$  and transmits  $x_{1,1}^R$  to U. In the meantime and similarly, V transmits  $x_{2,1}$  to BS  $y_B[2] = h_2 x_{1,1}^R + h_3 x_{2,1} + z_B[2]$ ,  $y_U[2] = h_1 x_{1,1}^R + h_4 x_{2,1} + z_U[2]$ . Third, if  $\mu \geq \lambda$ , RS transmits  $x_{1,2}^R$  to U interference-free  $y_U[3] = h_1 x_{1,2}^R + z_U[3]$ . If  $\mu < \lambda$ , V transmits  $x_{2,2}$  to BS interference-free  $y_B[3] = h_3 x_{2,2} + z_B[3]$ .

The total length of the transmissions for the direct user, which is the V–BS transmissions here, is pre-defined as  $\lambda N$  symbols. Denote the number of symbols in the RS–U transmissions as  $\mu N$ . Since BS and RS cannot transmit and receive at the same time, the BS–RS transmission cannot be performed simultaneously with any other transmission. Because the RS–U and V–BS transmissions do not completely coincide, the length of the BS–RS transmission is thus determined as  $(2 - \max(\mu, \lambda))N$  symbols. Therefore, the messages  $s_1$  and  $s_2$  are divided and encoded at RS and V respectively such that  $|x_{1,1}| = |x_{2,1}| = \min(\mu, \lambda)N$ . If  $\mu \geq \lambda$ ,  $|x_{1,1}^R| = |x_{2,1}| = |x_2|$  and  $|x_{1,2}^R| = \mu - \lambda$ . If  $\mu < \lambda$ ,  $|x_{2,1}| = |x_{1,1}^R| = |x_1^R|$  and  $|x_{2,2}| = \lambda - \mu$ . In the following, we estimate the optimal value of  $\mu$  for a pre-defined value of  $\lambda$ . Since BS knows  $x_1$  and therefore  $x_{1,1}^R$  and  $x_{1,2}^R$  thus BS cancels the contribution of  $x_1$  in the received signal. The total data sent through the BS–RS, RS–U and V–BS transmissions are respectively

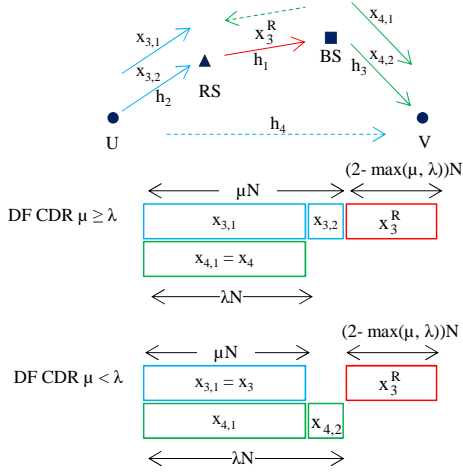


Fig. 2. Time slots in CDR Scheme  $S_2$ .

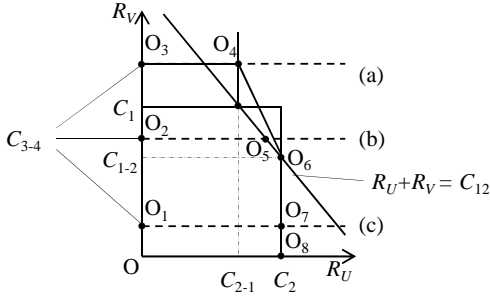


Fig. 3. Rate regions of  $(R_U, R_V)$  of  $S_2$  in slot 1.

$D_{U_1}^{S_1} = (2 - \max(\mu, \lambda))NC_1$ ,  $D_{U_2}^{S_1} = \min(\mu, \lambda)NC_{2-4} + (\mu - \min(\mu, \lambda))NC_2$ ,  $D_V^{S_1} = \lambda NC_3$ . The sum-rate of two users is  $R_S^{S_1} = \frac{D_S^{S_1}}{2N} = \frac{\min(D_{U_1}^{S_1}, D_{U_2}^{S_1}) + D_V^{S_1}}{2N}$ .

### C. CDR Scheme 2

First, U transmits  $x_{3,1}$  with rate  $R_U$  to RS and BS transmits  $x_{4,1}$  with rate  $R_U$  in  $\min(\mu, \lambda)N$  symbols simultaneously  $y_R[1] = h_2x_{3,1} + h_1x_{4,1} + z_R[1]$ ,  $y_V[1] = h_4x_{3,1} + h_3x_{4,1} + z_V[1]$ . Second, U transmits  $x_{3,2}$  to RS  $y_R[2] = h_2x_{3,2} + z_R[2]$  or BS transmits  $x_{4,2}$  to V  $y_V[2] = h_3x_{4,2} + z_V[2]$  in  $|\mu - \lambda|N$  symbols interference-free with maximal rates of the corresponding channels  $C_2$  and  $C_3$  respectively (see Fig. 2). Third, RS decodes  $x_{3,1}$  and  $x_{3,2}$ , re-encodes and forwards them to BS  $y_B[3] = h_1x_3^R + z_B[3]$ . Since BS and RS cannot transmit and receive at the same time, the RS-BS transmission cannot be performed simultaneously with any other transmission, it starts only after the first  $\max(\mu, \lambda)N$  symbols are finished. Thus  $|x_3^R| = |x_{3,1}^R| + |x_{3,2}^R| = (2 - \max(\mu, \lambda))N$ . We consider two cases:

- $C_1 < C_5$ :  $R_U$  and  $R_V$  are selected such that V can decode  $x_4$  treating  $x_3$  as noise  $R_V \leq C_{3-4}$  and RS can decode  $x_3$ . There are two cases to satisfy the second condition:
  - RS decodes  $x_3$  treating  $x_4$  as noise:  $R_U \leq C_{2-1}$ .
  - RS decodes both  $x_3$  and  $x_4$  according to Multiple Access Channel (MAC) [11]:  $R_U \leq C_2$ ,  $R_V \leq C_1$ ,  $R_U + R_V \leq C_{1,2}$ .

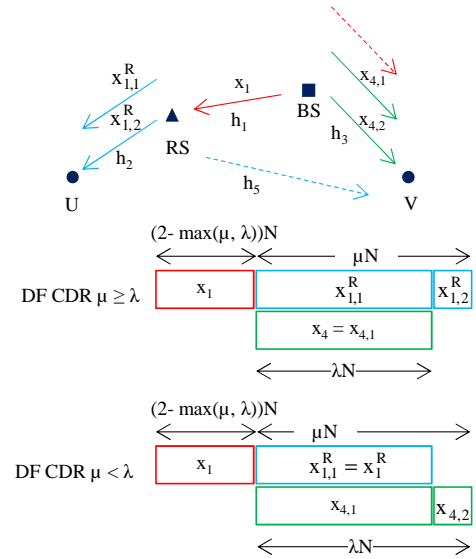


Fig. 4. Time slots in CDR Scheme  $S_3$ .

Fig. 3 demonstrates the rate region of  $(R_U, R_V)$ , the rates selected in slot 1. Depending on the value of  $C_{3-4}$  we have the region with different shapes: (a)  $OO_3O_4O_6O_8$  if  $C_{3-4} \geq C_1$ , (b)  $OO_2O_5O_6O_8$  if  $C_{1-2} \geq C_{3-4} < C_1$ , (c)  $OO_1O_7O_8$  if  $C_{3-4} < C_{1-2}$ .

- $C_1 \geq C_5$ : Here both BS and V can decode  $x_3^R$ . Using the information about  $s_3$ , the interference in the first slot at V can be completely canceled. Therefore in the first slot BS can transmit  $x_4$  to V with the maximal rate  $R_V \leq C_3$ . We have the same conditions as in the of  $C_1 < C_5$ , the only difference is that  $C_{3-4}$  is replaced by  $C_3$ . We have the same rate region of  $(R_U, R_V)$ , the rates selected in slot 1, as in Fig. 3 where  $C_{3-4}$  is replaced by  $C_3$ .

The data transmitted in U-RS, RS-BS, BS-V transmissions and the total data transmitted to two users are respectively  $D_{U_1}^{S_2} = \min(\mu, \lambda)R_UN + (\mu - \min(\mu, \lambda))C_2N$ ,  $D_{U_2}^{S_2} = (2 - \max(\mu, \lambda))C_1N$ ,  $D_V^{S_2} = \min(\mu, \lambda)R_VN + (\lambda - \min(\mu, \lambda))C_3N$  and  $D_S^{S_2} = \min(D_{U_1}^{S_2}, D_{U_2}^{S_2}) + D_V^{S_2}$ . The sum-rate is  $R_S^{S_2} = \frac{D_S^{S_2}}{2N}$ .

Above we consider the cases when V has to decode at least  $s_3$  or  $s_4$  in slots 1 and 2 or in slot 3. The case when V does not need to decode any of them can be achieved by using combining two replicas of the information sent originally by U, each encoded with a different codebook (one used by U and the other by RS). However, such a scheme is outside the scope of this paper.

### D. CDR Scheme 3

The transmissions are conducted in the following steps (Fig. 4): First, BS transmits  $x_1$  to RS in  $(2 - \max(\mu, \lambda))N$  symbols  $y_R[1] = h_1x_1 + z_R[1]$ . Second, RS and BS transmits  $x_{1,1}^R$  with rate  $R_U$  and  $x_{4,1}$  with rate  $R_V$  respectively and simultaneously in  $\min(\mu, \lambda)N$  symbols  $y_U[2] = h_2x_{1,1}^R + z_U[2]$ ,  $y_V[2] = h_5x_{1,1}^R + h_3x_{4,1} + z_V[2]$ . Third, RS transmits  $x_{1,2}^R$  in  $(\mu - \lambda)N$  symbols  $y_U[3] = h_2x_{1,2}^R + z_U[3]$  if  $\mu \geq \lambda$  and BS transmits  $x_{4,2}$  in  $(\lambda - \mu)N$  symbols  $y_V[3] = h_5x_{1,2}^R + h_3x_{4,2} + z_V[3]$  if  $\mu < \lambda$ . Note that when  $\mu \geq \lambda$ ,  $|x_{4,2}| = 0$  and  $x_{4,1} = x_4$

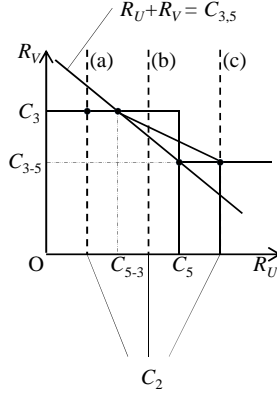


Fig. 5. Rate regions of  $(R_U, R_V)$  of  $S_3$  in slot 2.

and when  $\mu < \lambda$ ,  $|x_{1,2}| = 0$  and  $x_{1,1}^R = x_1^R$ . We consider two cases:

- $C_1 < C_3$ :  $R_U$  and  $R_V$  are selected such that U can decode  $x_{1,1}$  ( $R_U \leq C_2$ , since the BS-U channel is zero) and V can decode  $x_{4,1}$ . There are two cases to satisfy the second condition:
  - V decodes  $x_{4,1}$  treating  $x_{1,1}^R$  as noise:  $R_V \leq C_{3-5}$ .
  - V decodes both  $x_{1,1}^R$  and  $x_{4,1}$ :  $R_U \leq C_5$ ,  $R_V \leq C_3$ ,  $R_U + R_V \leq C_{3,5}$ .

The data transmitted in BS-RS, RS-U, BS-V transmissions and the total data transmitted for two users are respectively  $D_{U_1}^{S_3} = (2 - \max(\mu, \lambda))C_1N$ ,  $D_{U_2}^{S_3} = (\mu - \min(\mu, \lambda))C_2N + \min(\mu, \lambda)R_UN$ ,  $D_V^{S_3} = (\lambda - \min(\mu, \lambda))C_3N + \min(\mu, \lambda)R_VN$  and  $D_S^{S_3} = \min(D_{U_1}^{S_3}, D_{U_2}^{S_3}) + D_V^{S_3}$ . Similarly to Scheme 2, Fig. 5 demonstrates the rate region of  $(R_U, R_V)$ . It has different shapes corresponding to different values of  $C_2$ .

- $C_1 \geq C_3$ : Here RS and V can decode  $x_1$ . Using the information about  $s_1$ , the interference in slot 2 at V can be completely canceled. Therefore in slot 2, BS can transmit  $x_{4,1}$  to V with the maximal rate  $R_V = C_3$  while RS can transmit  $x_{1,1}$  to U with the maximal rate  $R_U = C_2$ . The data transmitted in RS-U, BS-V transmissions are different from the previous case  $D_{U_2}^{S_3} = \mu C_2N$ ,  $D_V^{S_3} = \lambda C_3N$ .

The sum-rate of  $S_3$  is  $R_S^{S_3} = \frac{D_S^{S_3}}{2N}$ . Again combining two symbol string with different codebooks can be used here at V to decode its desired signal.

#### E. CDR Scheme 4

The transmissions are conducted in the following steps (Fig. 6): First, U and V transmits  $x_{3,1}$  with rate  $R_U$  and  $x_{2,1}$  with rate  $R_V$  respectively and simultaneously in  $\min(\mu, \lambda)N$  symbols  $y_R[1] = h_2x_{3,1} + h_5x_{2,1} + z_R[1]$ ,  $y_B[1] = h_3x_{2,1} + z_B[1]$ . Second, U transmits  $x_{3,2}$  in  $(\mu - \lambda)N$  symbols  $y_R[2] = h_2x_{3,2} + z_R[2]$  if  $\mu \geq \lambda$  and V transmits  $x_{2,2}$  in  $(\lambda - \mu)N$  symbols  $y_B[2] = h_3x_{2,2} + z_B[1]$  if  $\mu < \lambda$  symbols. Third, RS transmits  $x_3^R$  to BS in  $(2 - \max(\mu, \lambda))N$  symbols  $y_B[3] = h_1x_3^R + z_B[3]$ . Note that when  $\mu \geq \lambda$ ,  $|x_{2,2}| = 0$  and  $x_{2,1} = x_2$  and when  $\mu < \lambda$ ,  $|x_{3,2}| = 0$  and  $x_{3,1} = x_3$ .

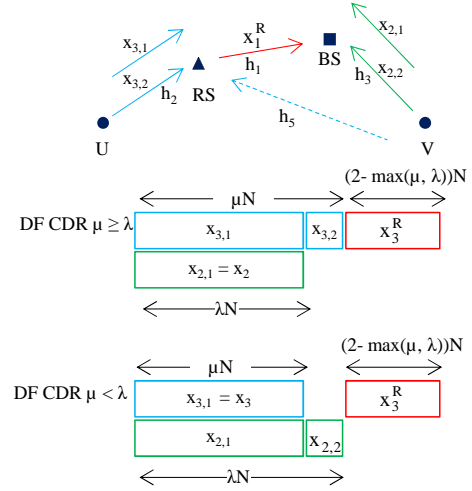


Fig. 6. Time slots in CDR Scheme  $S_4$ .

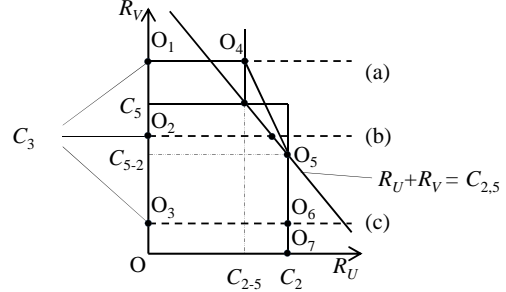


Fig. 7. Rate regions of  $(R_U, R_V)$  of  $S_4$  in slot 1.

$R_U$  and  $R_V$  are selected such that BS can decode  $x_{2,1}$  ( $R_V \leq C_3$ , since the U-BS channel is zero) and RS can decode  $x_{3,1}$ . There are two cases to satisfy the second condition:

- RS decodes  $x_{3,1}$  treating  $x_{2,1}$  as noise:  $R_U \leq C_{2-5}$ .
- RS decodes both  $x_{3,1}$  and  $x_{2,1}$ :  $R_U \leq C_2$ ,  $R_V \leq C_5$ ,  $R_U + R_V \leq C_{2,5}$ .

The rate region of  $(R_U, R_V)$  is demonstrated in Fig. 7. We have  $D_{U_1}^{S_4} = \min(\mu, \lambda)R_UN + (\mu - \min(\mu, \lambda))C_2N$ ,  $D_{U_2}^{S_4} = (2 - \max(\mu, \lambda))C_1N$ ,  $D_V^{S_4} = \min(\mu, \lambda)R_VN + (\lambda - \min(\mu, \lambda))C_3N$  and  $D_S^{S_4} = \min(D_{U_1}^{S_4}, D_{U_2}^{S_4}) + D_V^{S_4}$ . The sum-rate of  $S_4$  is  $R_S^{S_4} = \frac{D_S^{S_4}}{2N}$ .

#### IV. NUMERICAL RESULTS

Fig. 8 shows the rate regions  $(R_U^i, R_V^i)$ ,  $i \in \{S_1, S_2, S_3, S_4, E\}$  of different schemes, where  $R_j^i$ ,  $j \in \{U, V\}$ , is the rate delivered to user  $j$  in scheme  $i$ . The simulation is conducted in case of channels  $\bar{\gamma} = [\gamma_1 \ \gamma_2 \ \gamma_3 \ \gamma_4 \ \gamma_5] = [15 \ 10 \ 13 \ -10 \ 0]\text{dB}$ . The simulation result of rate regions is achieved by calculating the rate pair  $(R_U^i, R_V^i)$  for all values of  $\lambda$ ,  $\mu$  and  $R_U$ ,  $R_V$  which are selected such that satisfying the conditions in each scheme with resolution  $\Delta\lambda = \Delta\mu = 0.1$  and  $\Delta R_U = \Delta R_V = 0.2$ . The reference scheme has the most contained rate region since it does not exploit the information about the interference as all of the CDR schemes do. CDR scheme 1 has best rate region (high  $R_V$  and not low  $R_U$ ) because the only limiting factor in this scheme is the interference from V to U over the inter-user channel,



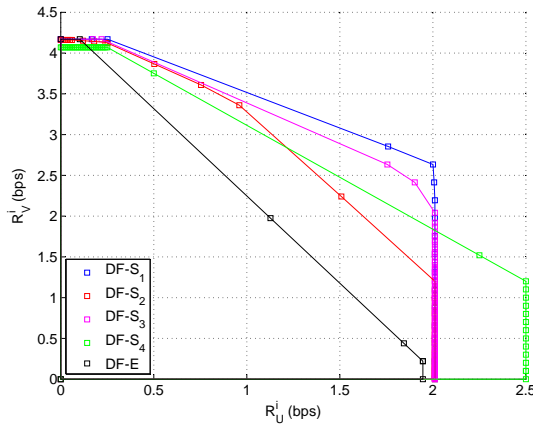


Fig. 8. Rate region of  $(R_U^i, R_V^i)$  ( $i \in \{S_1, \dots, S_4, E\}$ ) for DF CDR and reference schemes.

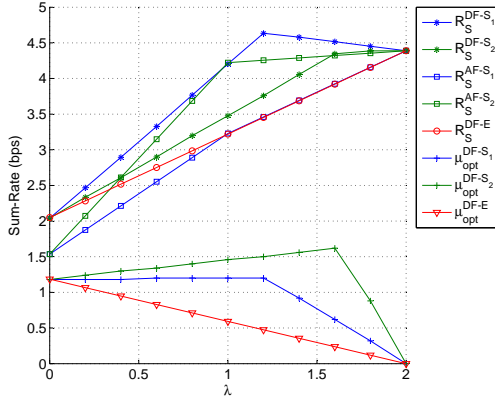


Fig. 9. Sum-rate and optimal  $\mu$  for CDR Scheme 1 and 2 and reference scheme. Note that  $\mu$  has a different unit.

however, this channel is chosen as low (-10dB). Actually, this assumption is viable because in a cellular network with relays, the channel between two users is normally lower than the channel between a user and a infrastructure station (RS, BS) or the channel between two infrastructure stations (they are designed in good positions). Moreover, relayed users certainly appear in the region, which is covered by the RSs, far from the region where the direct users appear which is cover by the BS.

Fig. 9 and 10 show sum-rate of DF and AF CDR scheme 1,

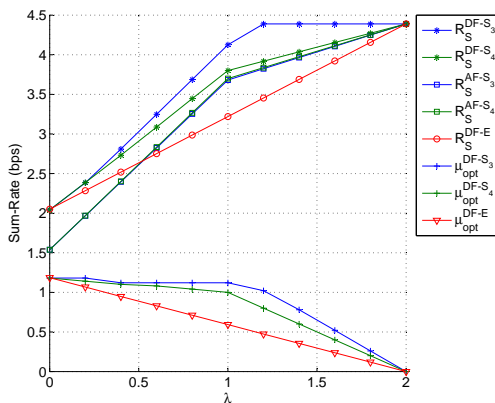


Fig. 10. Sum-rate and optimal  $\mu$  for CDR Scheme 3 and 4 and reference scheme. Note that  $\mu$  has a different unit.

2, 3, 4 and reference DF scheme when  $\lambda$  varies. The channels are selected as in the previous simulation. With a specific value of  $\lambda, \mu, R_U$  and  $R_V$  are selected such that the sum-rate achieve the maximal value. All schemes achieve a higher sum-rate when  $\lambda > 1$  and goes to 2 because the time resource is given to a direct transmission is more efficient than to a relayed transmission. Most of the schemes have a non-decreasing sum-rate with  $\lambda$ . Only DF CDR scheme 1 achieve the maximal value when  $\lambda < 2$ . This is because this scheme well exploit the information of the interference as explained above. Most of DF CDR schemes are always better than AF CDR schemes except that DF CDR scheme 2 is worse than AF CDR scheme 2 when  $\lambda$  is medium. This is because all the interference received in AF CDR schemes is used to decode the desired signal using MMSE while in DF CDR scheme 2, RS has to receive the desired signal and the interference using MAC which limits the two rates.

## V. CONCLUSION

In this paper, we propose and analyze the Coordinated transmissions to Direct and Relayed user in a wireless cellular network with relays using Decode-and-Forward. The durations of the transmissions for the direct and relayed users as well as the rates of simultaneous transmissions are optimized to have the best rate region and the maximal sum-rate. We compare the quality of the proposed schemes with their version of Amplify-and-Forward as well as the conventional scheme. Numerical results show that the proposed schemes almost provide better rates and sum-rate than the AF and reference schemes.

## ACKNOWLEDGMENT

This work is supported by the Danish Research Council for Technology and Production, grant nr. 09 – 065035.

## REFERENCES

- [1] G. Farhadi and N. C. Beaulieu, "Capacity of amplify-and-forward multi-hop relaying systems under adaptive transmission," *IEEE Trans. on Comm.*, vol. 58, iss. 3, pp. 758-763, 2010.
- [2] Y. Zhu, P.-Y. Kam, and Y. Xin, "Differential modulation for decode-and-forward multiple relay systems," *IEEE Trans. on Comm.*, vol. 58, iss. 1, pp. 189-199, 2010.
- [3] Z. Liu, M. Uppal, V. Stankovic, and Z. Xiong, "Compress-Forward Coding With BPSK Modulation for the Half-Duplex Gaussian Relay Channel," *IEEE Trans. on Signal Processing*, vol. 57, iss. 11, pp. 4467 - 4481, 2009.
- [4] P. Popovski and H. Yomo, "Bi-directional Amplification of Throughput in a Wireless Multi-Hop Network," *IEEE VTC*, Spring 2006.
- [5] S. Katti, S. Gollakota, and D. Katabi "Embracing Wireless Interference: Analog Network Coding," *ACM SIGCOMM*, 2007.
- [6] H. Ning, C. Ling, and K. K. Leung, "Wireless Network Coding with Imperfect Overhearing," *arXiv:1003.4270v1 [cs.IT]* 22 Mar 2010.
- [7] C. Thai and P. Popovski, "Coordinated Direct and Relay Transmission with Interference Cancellation in Wireless Systems," *IEEE Comm. Letters*, vol. 15, no. 4, April 2011, pp. 416-418.
- [8] C. Thai, P. Popovski, M. Kaneko and E. Carvalho, "Coordinated Transmissions to Direct and Relayed Users in Wireless Cellular Systems," in *Proc. IEEE ICC*, Kyoto, Japan, Jun 2011.
- [9] C. Thai and P. Popovski, "Coordination of Regenerative Relays and Direct Users in Wireless Cellular Networks," to appear in *Proc. IEEE ISWCS'11*, Aachen, Germany, Nov 2011.
- [10] H. Cho and J. Andrews, "Resource-redistributive opportunistic scheduling for wireless systems," *IEEE Trans on Wireless Communications*, vol. 8, pp. 3510 - 3522, July 2009.
- [11] D. Tse and R. Viswanath, "Fundamentals of Wireless Communications," Chap. 8, Cambridge Press, 2005.